Appendix I

Accelerating ESATAN-TMS Thermal Convergence for Strongly Coupled Problems

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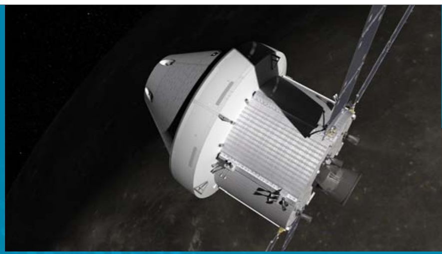
Abstract

ESATAN-TMS Thermal solves the heat conductance differential equation (DE) for the lumped parameter thermal network node temperatures considering heat sources as well as linear (also one way) and quartic (radiative) heat exchanges between the nodes. Extensions to this modeling are available for fluid loops and ablation, namely FHTS and ABLAT. However, embedding other relevant thermodynamic phenomena, as e.g. ice sublimation during ascent of a launcher or pressurization/depressurization of a vessel, may provoke other strongly coupled heat sources and additional, segregated DE, which may impact the accuracy of the result. Even then one will usually succeed in reaching the required accuracy by choosing sufficient small time-steps, but at the cost of significantly increased CPU time. An innovative method based on a predictor-corrector-method (PCM), representing a workaround for accelerating the convergence, has been implemented and will be explained here. This method uses standard ESATAN entities only, i.e. auxiliary nodes, heat sources and one way linear conductors. For the example of ice sublimation during launcher's ascent this method is explained in detail and the benefit is demonstrated in conjunction with a specific solver option provided by ESATAN-TMS Thermal software developers in the frame of this work. Using this innovative method the time-step can be increased by nearly a factor of 100 for the given example.



Accelerating ESATAN's Convergence for Strongly Coupled Problems

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03.11.2015

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Agenda

- Introduction
- Proposed Predictor-Corrector Method (PCM)
- Demonstration case: Ice sublimation
- Improvement for ice sublimation induced by the method
- Conclusion and next steps

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Introduction

- Problem
  - Strongly coupled heat sources and additional, segregated Differential Equations (DE) within ESATAN (which solely solves for the heat conductance DE) may impact the accuracy of the results
  - Usually, sufficient small time-steps succeed in reaching the required accuracy, BUT at the cost of CPU time
  - Thus, ways have been studied to accelerate ESATAN's convergence for a required time-step
  - Example outlined here is for ice sublimation during launcher's ascent (strong coupling comes from huge amount of latent heat of ice sublimation of about 3 MJ/kg)

- Methods studied
  - Enabling SLCRNC, benefit from control-constant METHOD has been systematically studied:
    - METHOD=0 (default): $VARIABLES1 is called once for the forward step and twice for the backward step
    - METHOD=2: $VARIABLES1 is called at every iteration, BUT at the cost of CPU time
  - Workaround to accelerate convergence by linear approximation (predictor-corrector-method PCM) is based on ESATAN basic features:

$$Y = Y|_{T^{old}} + \frac{\partial Y}{\partial T}\bigg|_{T^{old}} \delta T$$

$$where\ \delta T = T - T^{old}\ \ is\ the\ difference\ of\ temperature\ T^{old} at\ \$VARIABLES1\ access$$
$$and\ the\ current\ temperature\ T\ of\ the\ iteration$$

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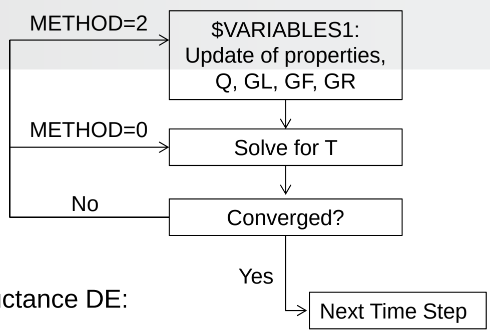
Proposed PCM



Recall of the general Lumped Parameter heat conductance DE:

$$C_i \frac{dT_i}{dt} = Q_i + \sum_{j \neq i} GL(T_j, T_i)(T_j - T_i) + \sum_{j \neq i} GF(T_j, T_i)(T_j - T_i) + \sum_{j \neq i} GR(T_j, T_i)(Tj^4 - Ti^4)$$

PCM for an affected temperature node:

$$C \frac{dT}{dt} = Q + GF(T^{old}, T)(T^{old} - T)$$

where:

T^{old}	temperature at $VARIABLES1 access
T	current temperature of the iteration
$Q = f(..., T^{old})$	heat source
$GF(T^{old}, T) = -\frac{\partial f}{\partial T}$	

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Proposed PCM, cont'd

PCM for an affected auxiliary variable node with C=0 (arithmetic node):

$$0 = Q + GF(0,X)(0 - X) + GF(\delta T/2, X)(\delta T/2 - X) \qquad \Rightarrow X = X^{old} + \frac{\partial X}{\partial T}\delta T$$

where:
X auxillary variable

$\delta T/2 = (T^{old} - T)/2$
derived from an arithmetic node containing $\delta T/2$*:*
$0 = GF(T, \delta T/2)(T - \delta T/2) + GF(T^{old}, \delta T/2)(T^{old} - \delta T/2)$
with $GF(T, \delta T/2) = GF(T^{old}, \delta T/2) = 1$

$$Q = X^{old}\left(1 + \left(\frac{1}{2\frac{\partial X}{\partial T}} - 1\right)^{-1}\right)$$

$$GF(0, X) = 1$$

$$GF(\delta T/2, X) = \left(\frac{1}{2\frac{\partial X}{\partial T}} - 1\right)^{-1}$$

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Proposed PCM: Explanation of $\delta T/2$ Calculation-Scheme

D1 : T1	*current ice surface temperature of the iteration*
$B2: T2 = -T^{old}$	*at $VARIABLES1 access*
D3 : T3	*half of temperature difference, C3=0*

0=GF(1,3) (T1-T3) + GF(2,3) (T2-T3)
with
GF(1,3) = 1
GF(2,3) = 1

$\Rightarrow$ *T3=(T1+T2)/2*
$\widehat{=}$ $T3 = (T - T^{old})/2$

$\Rightarrow$ T3= δT/2

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Proposed PCM: Explanation of X Calculation-Scheme

D1: T1=X *auxiliary variable, C1=0*
B2: T2=0
D3: T3=δT/2 *half of temperature difference, C3=0*

0=Q1 +GF(2,1) (T2-T1) + GF(3,1) (T3-T1)
with

$Q1 = X^{old}\left(1+\left(\frac{1}{2\frac{\partial X}{\partial T}}-1\right)^{-1}\right)$ *at $VARIABLES1 access*

GF(2,1) = 1

$$GF(3{,}1) = \left(\frac{1}{2\frac{\partial X}{\partial T}}-1\right)^{-1}$$

$$\Rightarrow 0 = X^{old}\left(1+\left(\frac{1}{2\frac{\partial X}{\partial T}}-1\right)^{-1}\right) - T1 + \left(\frac{1}{2\frac{\partial X}{\partial T}}-1\right)^{-1}(T3-T1)$$

$$\Leftrightarrow T1\left(1+\left(\frac{1}{2\frac{\partial X}{\partial T}}-1\right)^{-1}\right) = X^{old}\left(1+\left(\frac{1}{2\frac{\partial X}{\partial T}}-1\right)^{-1}\right) + \left(\frac{1}{2\frac{\partial X}{\partial T}}-1\right)^{-1} T3$$

$$\Leftrightarrow T1 = X^{old} + \left(\frac{1}{2\frac{\partial X}{\partial T}}-1\right)^{-1} \Big/ \left(1+\left(\frac{1}{2\frac{\partial X}{\partial T}}-1\right)^{-1}\right) T3$$

$$\Rightarrow T1 = X^{old} + \frac{\partial X}{\partial T}\delta T$$

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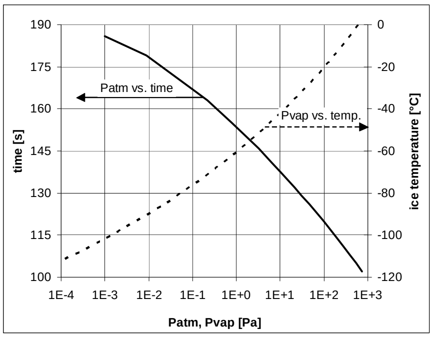
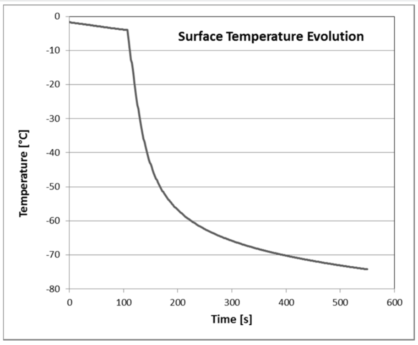
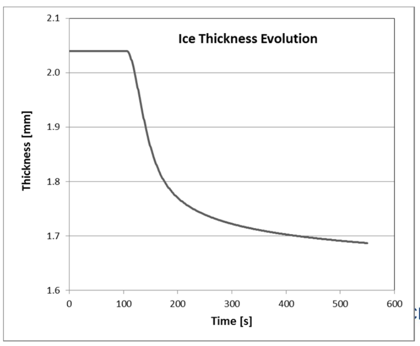
Ice Sublimation during Ascent: Description







$$\dot{m} = \frac{\gamma \cdot (p_{vap} - p_{amb})}{\sqrt{\frac{2 \cdot \pi \cdot \Re \cdot T}{M_{H_2O}}}}$$ Ice sublimation flow rate (Hertz-Knudsen) [kg/s/m²]

$$Q = A \cdot \dot{m} \cdot H$$ Ice cooling rate [W]

$$\dot{\tau} = \frac{\dot{m}}{\rho}$$ Ice thickness reduction rate [m/s] (additional, segregated DE)

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Ice Sublimation during Ascent: Implementation of PCM

Ice surface temperature T-PCM:

$$C\frac{dT}{dt} = Q + GF(T^{old}, T)(T^{old} - T)$$

where:
$Q = A\ \dot{m}^{old} H$
$GF(T^{old}, T) = -H\frac{\partial \dot{m}}{\partial T}$

Ice sublimation rate M-PCM:

$$0 = Q + GF(0,\dot{m})(0 - \dot{m}) + GF(\delta T/2,\dot{m})(\delta T/2 - \dot{m})$$
$$\Rightarrow \dot{m} = \dot{m}^{old} + \frac{\partial \dot{m}}{\partial T}\delta T$$

where:
$GF(0,\dot{m}) = 1$
$$Q = \dot{m}^{old}\left(1 + \left(\frac{1}{2\frac{\partial \dot{m}}{\partial T}} - 1\right)^{-1}\right)$$
$$GF(\delta T/2,\dot{m}) = \left(\frac{1}{2\frac{\partial \dot{m}}{\partial T}} - 1\right)^{-1}$$

Ice vapor pressure P-PCM:

$$0 = Q + GF(0,p)(0 - p) + GF(\delta T/2,p)(\delta T/2 - p)$$
$$\Rightarrow p = p^{old} + \frac{\partial p}{\partial T}\delta T$$

where:
$GF(0,p) = 1$
$$Q = p^{old}\left(1 + \left(\frac{1}{2\frac{\partial p}{\partial T}} - 1\right)^{-1}\right)$$
$$GF(\delta T/2,p) = \left(\frac{1}{2\frac{\partial p}{\partial T}} - 1\right)^{-1}$$

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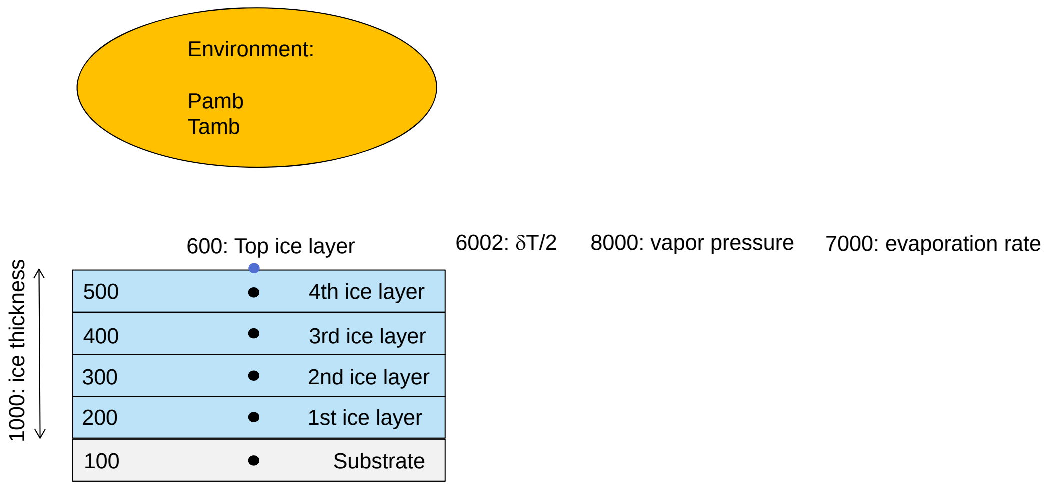
Ice Sublimation during Ascent: Network



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Ice Sublimation during Ascent: Cases

Duration: 0 … 550s
Required time resolution: 1s

Time steps: 0.0001s (reference)
0.01s
0.1s
0.5s
1s

SLCRNC	Convergence Acceleration		
METHOD=0	NOMINAL (w/o PCM)	T-PCM	TMP-PCM
METHOD=2	NOMINAL (w/o PCM)	T-PCM	TMP-PCM

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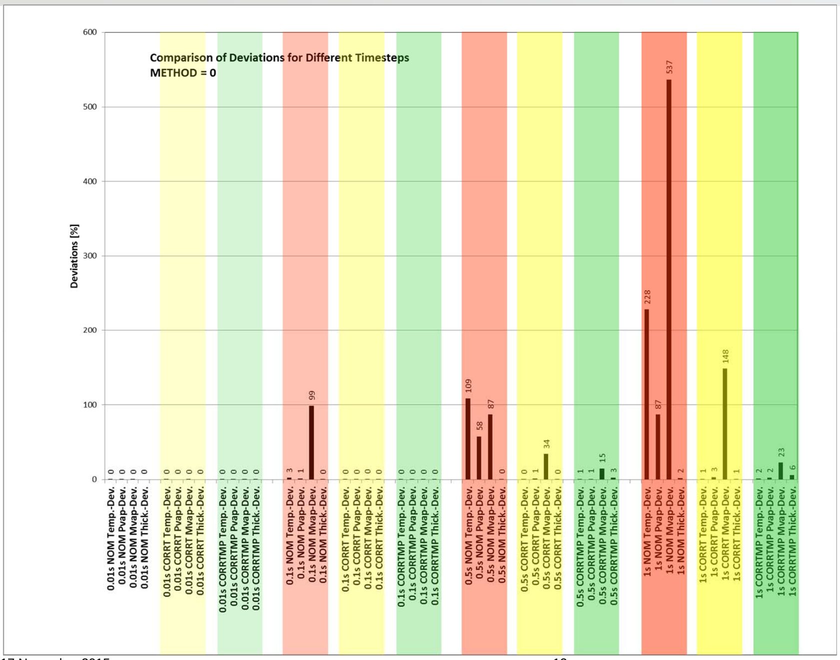
Ice Sublimation during Ascent: Max Deviations for METHOD=0



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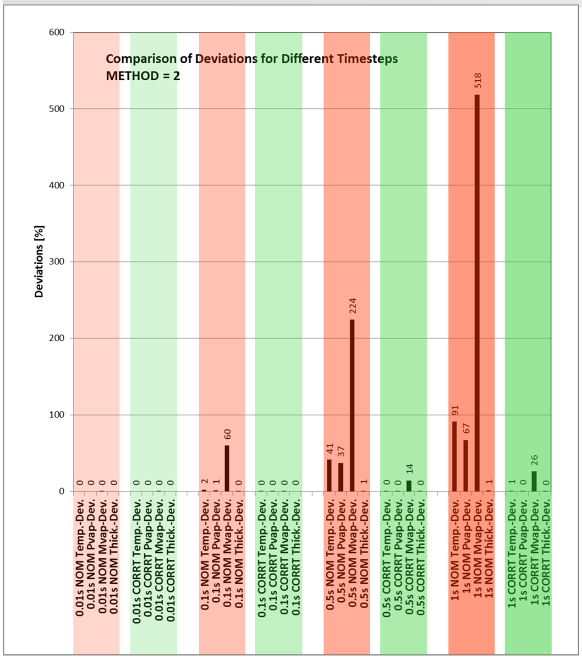
Ice Sublimation during Ascent: Max Deviations for METHOD=2



No significant benefit from *M-PCM* und *P-PCM correction*

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Conclusion

- PCM workaround for strongly coupled and additional DE generally explained:
  - PCM for temperature node
  - PCM for auxillary variable node (airthmetic)

- Example given concerning ice sublimation during ascent:
  - SLCRNC with METHOD=0 and METHOD=2 studied
  - Ice surface temperature correction: T-PCM
  - Ice sublimation rate correction: M-PCM (arithmetic)
  - Ice vapor pressure correction: P-PCM (arithmetic)
  - Ice thickness reduction DE

  ➢ Exact results are obtained without PCM with a time-step of 0.01s
  ➢ With a time-step of 1s correct results have been achieved
    ➢ For METHOD=0 with TMP-PCM
    ➢ For METHOD=2 with T-PCM (M-PCM and P-PCM in addition give no significant improvement)

➢ Need for dedicated System Elements and a general way to couple more thermodynamic equations to ESATAN's heat conductance DE,
as e.g. ice sublimation, pressurization / depressurization of a vessel, …

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