Appendix O

Rationalisation of Stabilisation Criteria for Thermal Balance Tests

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Abstract

When a space vehicle or its lower level elements are subjected to thermal testing, it is recurrently found that concepts such as "thermal equilibrium" and "thermal stability" are addressed lacking necessary rigor. Therefore, the quality of the results of possibly expensive Thermal Balance Tests may be diminished, if not jeopardized.

The set of concepts developed and presented are meant to allow thermal specialists to deal with stabilisation aspects of thermal balance tests on a rational ground. Moving from the theory of thermal transient analyses carried-out by means of lumped-parameter network mathematical models and using Linear Algebra, the behaviour of a network close to stabilisation is studied. The novel concepts of "instantaneous" time constant and network "terminal" time constants are introduced. The latter, in particular, is shown to be a powerful means sufficient to describe the whole network nearly-stable behaviour.

Methods to determine the terminal time constant of complex networks are illustrated, and crucial conclusions of practical interest are drawn on the ability to keep by simple means under strict control the errors arising from the truncation of stabilisation transients.

Rationalisation of Stabilisation Criteria for Thermal Balance Tests

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Introduction – *Background Overview*

Well known steps typically involved in thermal design and verification of Spacecraft, Subsystems, Sub-Assemblies, Equipment (etc.) include:

- Establishment of a TMM (Thermal Mathematical Model)
- Preliminary utilisation of that TMM for design purposes
- Validation (fine tuning) of TMM by means of correlation with a Thermal Balance Test, for which test predictions are produced
- Production of on-orbit thermal predictions

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Introduction – *Background Overview*

Thermal Balance Tests, irrespective of the dimensions and complexity of the test article involved are recurrently judged

- Too long, by Project Managers and AIT Managers
- Too short and with too few test phases, by Thermal Analysts and Thermal Engineers

While, on the other hand it is indisputable that:

- Costs are normally relatively high and very steeply increasing with duration are indisputable
- *A variety of approaches and standards dealing with "Stabilisation Criteria" are in use, not always easy to defend, whereas they have obvious effects on Thermal Test duration*

Therefore:

- ***A solid and rational approach to trade-off test duration (and costs) against accuracy of results is more than just desirable.***

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Introduction – *Preliminary Considerations*

By common sense, an ideal Thermal Stabilisation Criterion:

- ***Should be solely based on engineering considerations about "affordable truncation errors" (i.e.: conscious acceptance of differences between actual instantaneous temperature measurements vs their true asymptotic values)***

Moreover, for practical reasons:

- ***It should involve only measurable quantities (temperatures, time)***
- ***It should have simple and straightforward formulation, verifiable in real time during test execution*** *(even by half-asleep engineers)*

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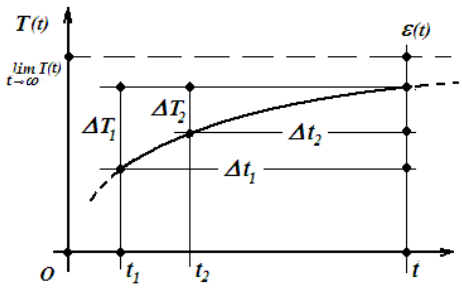
Introduction – *Preliminary Considerations*

Trivial starting point: single diffusive node linear thermal network:

- Temperature evolution governed by known Equation type: $T(t) = a + be^{-t/\tau}$
- Consequent Equation of truncation error can be easily be determined, without involving the unknown parameters a and b, as:

$$\varepsilon(t) = \frac{|\Delta T_1|}{e^{\Delta t_1/\tau} - 1} = \frac{|\Delta T_2|}{e^{\Delta t_2/\tau} - 1}$$



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Introduction – *Preliminary Considerations*

Noticeable facts, *once the thermal time constant "τ" of the network is known*:

1. Truncation error can be determined (precisely, in this case) by mere reliance on observable (measurable or predictable) parameters (temperature and time)
2. Given an acceptable truncation error ε a (virtually) arbitrary choice can be made among consistent couples of ΔT 's and Δt's to verify – at a given time – that truncation error is matched (or not exceeded).

Remarkably, virtually identical considerations apply when a generic non-linear thermal network (of whatever number of nodes and conductors) is dealt with. In particular:

- ***A process will be illustrated to extend these basic concepts, by means of identifying one single network-specific parameter (the "Terminal Time Constant") capable to synthesize the network response***
- ***The need for a relatively modest computational efforts will be shown to quantify the Terminal Time Constant of networks of whatever complexity.***

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Introduction – *Development Overview*

Line of thinking to attain the anticipated results:

- Acknowledge that the thermal transient of any system eventually approximate the behaviour of linear system
- Adopt an operative definition of a time-dependent pseudo time-constant to establish the legitimacy of the linear approximation
- Capitalise on the Physical fact that eventually all temperatures evolve along exponential curves characterised by **the same (terminal) time constant**
- Investigate ways to determine the value of the terminal time constant (the only non-measurable quantity encountered within this context)
- Illustrate the outcome of Linear Algebra utilisation: the maximum (negative) Eigenvalue method
- Propose the Discrete Derivation method based on the novel definition of "Instantaneous Time Constant"
- Extend the concept of truncation error control to complex networks, as a means to preserve compliance to the postulated best common sense approach
- Suggest rationalised approach to stabilisation criteria for practical application

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Introduction – *Development Overview*

Further considerations:

- Assessment of the effect of stabilisation criteria on test duration
- Warnings about the concept of "local time constant"

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Note:

Expansion and deepening of the subject presented, including legitimacy of approximations, are among the matters dealt with in my previous paper:

"Thermal Balance Testing: A Rigorous Theoretical Approach to Stabilisation Criteria Based on Operative Re-Definition of Thermal Time Constant" (AIAA Publication 2012-3405, 42nd ICES, S. Diego, CA – July 2012) Copyright © 2012 by Ettore Colizzi - ESA/ESTEC.

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Considerations about Nearly-Stable Network Behaviour

- Irrelevance of knowledge about network thermal evolution history
- Modest amplitude of temperature changes allows linearisation of the description of the network behaviour and legitimates reliance of linear algebra

  Note: *this encompasses any sources of non-linearity (e.g.: material property dependence on temperature) , and not only the most obvious radiation exchange mechanism*
- Capitalising from the time constant property of truly linear networks:

$$\tau = -\frac{dT_1}{dt} \Big/ \frac{d^2T_1}{dt^2} = -\frac{d^nT_1}{dt^n} \Big/ \frac{d^{n+1}T_1}{dt^{n+1}} = constant$$

by analogy, a "pseudo time constant", function of time, can be generically defined as an ad-hoc analytical tool to "gauge" the network actual proximity to linear behaviour:

$$\tau^*(t) = -\frac{dT_1}{dt} \Big/ \frac{d^2T_1}{dt^2}$$

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The Eigen-vector Method – *Algebra*

Briefly recalling from linear algebra: the N equation of energy balance at each of the N nodes:

$$c_n \frac{dT_n}{dt} = p_n - \sum_{q=1}^{N} K_{n,q}(T_n - T_q) - \sum_{b=1}^{N_b} H_{n,b}(T_n - \theta_b)$$

are resolved by means of Eigen-vector and Eigen value method. I.e: by resolving the set of simultaneous equations (in vector form below):

$$\frac{d}{dt}\vec{T} = [A]\,\vec{T} + \vec{F}$$ (Complete Equation)

$$[A]\,\vec{U} = \lambda\,\vec{U}$$ (Eigen-value and Eigen-vector Generator)

$$0 = [A]\,\vec{V} + \vec{F}$$ (Auxiliary Equation for Particular Integral Determination)

and imposing initial conditions. The latter operation allows then to determine the values of the N coefficients b_k and the complete final solution (vector and scalar form, respectively):

$$\vec{T}(t) = \vec{V} + \sum_{k=1}^{N} b_k \vec{U}_k e^{\lambda_k t} \qquad T_n(t) = v_n + \sum_{k=1}^{N} b_k u_{k,n} e^{\lambda_k t}$$

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The Eigen-vector Method – *Existence of the Terminal Time Constant*

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Linear algebra allows to demonstrate that:

- *All network nodal temperature profiles are a linear combination of the same family of (decaying) exponential functions*

Therefore**, after sufficiently long time:**

- *Only one (most persistent) exponential will prevail on all the other as a descriptor of the evolution of all nodes thermal behaviour*

As such:

- *All residual transient profiles will be characterised by the same* ***terminal time constant*** τ_∞ *associated to the* ***(unknown)*** *Eigen-value* λ_m *characterised by minimum absolute value, with:*

$$\tau_\infty = -\frac{1}{\lambda_m}$$

- *The equation of the nodal temperature will approximately be (as for the one-diffusive node network case):*

$$T_n(t) \approx v_n + b_m u_{m,n} e^{\lambda_m t} = v_n + \beta_n e^{-t/\tau_\infty}$$

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The Eigen-vector Method – *Terminal Time Constant Calculation*

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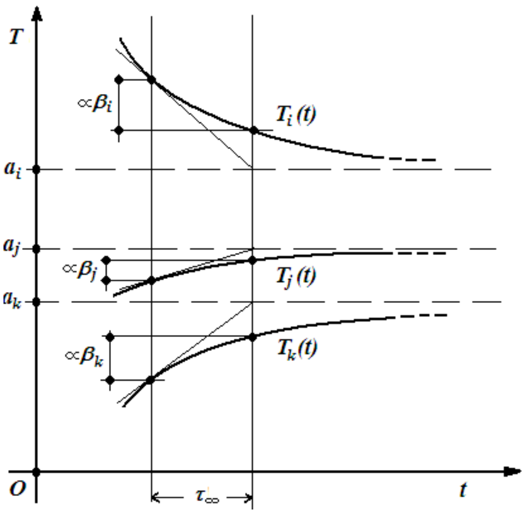
Further effects of the existence of a terminal time constant:

- the rate of change of all temperatures and components of the (terminal) Eigenvector $\vec{U}_m$ satisfy the equalities:

$$\frac{dT_i(t)}{dt} \bigg/ \frac{dT_j(t)}{dt} \approx \frac{T_i(t) - T_i(t-\eta)}{T_j(t) - T_j(t-\eta)} \approx \frac{\beta_i}{\beta_j} = \frac{u_{m,i}}{u_{m,j}}$$

- Once the (terminal) Eigen-vector is known, the semi-arbitrary choice of η allow to attain the explicit expression:

$$\lim_{t\to\infty} \frac{\sum_{s=1}^{N} \left[T_s(t) - T_s(t-\eta)\right] c_s}{\sum_{s=1}^{N} \left[T_s(t) - T_s(t-\eta)\right] \sum_{b=1}^{N_b} H_{s,b}} = \tau_\infty$$



Asymptotic Behaviour of Temperatures Under Terminal Time-Constant Regime

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The Eigen-vector Method – *Implementation*

The How to make use of these findings?

1. Establish a value for η such that expected temperature differences surpass (with margins) the numerical accuracy of the test predictions about to be carried-out
2. Implement an automated process in the TMM for test article external radiative conductor linearisation
3. Implement automatic algorithm in the TMM network solver carrying-out the summations over the entire set of diffusive nodes of the network (numerator) and double summation over diffusive note and boundary nodes (denominator)
4. Produce test predictions for the transient leading to stabilisation: the output will include by default $\tau_\infty(t)$
5. Check convergence of $\tau_\infty(t)$ to an asymptotic value
6. Adopt $\tau_\infty(t)$ (asymptotic value) for transient truncation control strategy
7. Make use of the temperature measurements taken during the test (and without any need of the TMM) to confirm, if necessary, the terminal time constant value theoretically determined

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Discrete Differentiation Method - *Principle*

- **Principle: direct application of the definition of $\tau^*(t)$** :

$$\tau^*(t) = -\frac{dT_1}{dt} \Big/ \frac{d^2T_1}{dt^2}$$

by numerical differentiation process carried-out by the TMM network solver during the Thermal Balance test prediction runs, *on the temperature profile of any diffusive node*

Example of numerical differentiation algorithm for equally spaced time intervals:

$$\tau^*(t) = -\frac{dT(t)}{dt} \Big/ \frac{d^2T(t)}{dt^2} \approx -\left(\frac{T_3 - T_1}{2\Delta t}\right) \Big/ \left(\frac{T_3 + T_1 - 2T_2}{\Delta t^2}\right) = -\frac{\Delta t}{2} \frac{T_3 - T_1}{T_3 + T_1 - 2T_2}$$

Warning: meaningful results will be conditional to the satisfaction of the minimum conditions safeguarding against numerical differentiation error, i.e.:

$$\Delta t > 8\tau \left(\frac{\delta T}{|\Delta T|}\right) \Big/ \left(\frac{\delta \tau}{\tau}\right)$$

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Discrete Differentiation – *Implementation*

The method is node-specific, therefore:

- *It does not need to be applied to the whole population of diffusive nodes (ideally any-one of the diffusive nodes would have to be sufficient to provide meaningful results!)*

As a **reasonable compromise**:

- *Use of a small population of pilot nodes of various typology scattered over the test article is recommended*

Expected results, and **demonstration of process consistency** will be obtained by observing:

- *Convergence of $\tau^*(t)$ to an asymptotic value, for each of the nodes*
- *Strong trend to coincidence of all the asymptotes of all pilot nodes to the same numerical (time) value, i.e.: the network terminal time-constant)* $\tau_\infty(t)$

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Stabilisation Criteria – *Truncation Error Control*

Main implication of the ability to determine network τ_∞ :

- Ability to transfer to any networks of whatever complexity the considerations made about the single diffusive node network (ideally, by simply replacing τ by τ_∞)
- Assurance that in a large network for each diffusive node *j*, future deviations of its temperature from its asymptotic value will be such to satisfy the inequality:

$$\varepsilon_j(t) < \frac{|\Delta T_j|}{e^{\Delta t/\tau_\infty} - 1} = \varepsilon_{j,MAX}$$

I.e.: ***If an upper limit is imposed to the maximum temperature drifts observed during a TB test over a relatively recent past (measured in terminal time constant units) the maximum expected deviation form their asymptotes will be constrained within a known quantity.***

Note: This matches the goals of simple formulation and utilisation of directly measured parameters only

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Truncation Error Control: *Practical Considerations*

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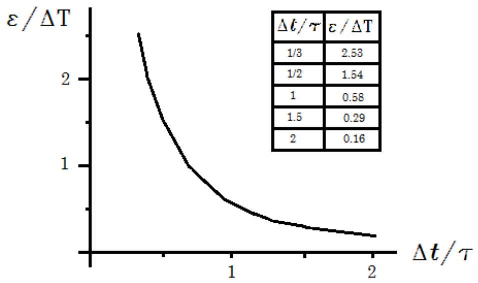
- Interest in shortening the observation time, to avoid unnecessarily prolong the test

On the other hand:

- Observed temperature differentials, well in excess of the accuracy of temperature measurements are desirable

Therefore, in conclusion

- ***Systematic utilisation of the illustrated rational approach is highly recommended***
- ***Adaptation to specific cases should be traded-off within the offered degree of freedom***



$\Delta t/\tau$	$\varepsilon/\Delta T$
1/3	2.53
1/2	1.54
1	0.58
1.5	0.29
2	0.16

Granted asymptote proximity as a function of observation time

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Truncation Error Control: *Effect on Test Duration*

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Further relationships assisting the selection of Stabilisation Criterion:

- **Minimum observation time** for a given affordable truncation error

$$\Delta t > \tau_\infty \ln\left(1 + \frac{|\Delta T|}{\varepsilon_{j,MAX}}\right)$$

Limiting factor: *minimum value* $|\Delta T|$ *of reliable temperature differential measurements over time*

- **Duration of Steady State test phases** as a function of allowed truncation error

$$D(\varepsilon^{(2)}_{j,MAX}) = D(\varepsilon^{(1)}_{j,MAX}) - \tau_\infty \ln \frac{\varepsilon^{(2)}_{j,MAX}}{\varepsilon^{(1)}_{j,MAX}}$$

Note: *only duration increment/decrement attributable to different truncation error can be quantified, since possibly lengthy transients between thermally distant levels are obviously test-specific.*

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Eigenvalues and Local Time Constants

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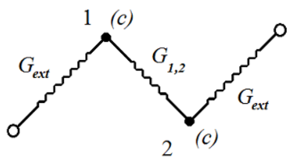
A word of warning:

- Network Eigenvalues (and associated time constants) are characteristic of the entire diffusive node network
- "Local" time constants associated to the N diffusive node taken one by one depend on network parameters too



But:

No actual direct relationship exists between magnitude of local time constants, network Eigenvalues and network terminal Time constant

$$G_{1,2} >> G_{ext}$$

$$\tau_1^{(Local)} = \tau_2^{(Local)} \sim \frac{c}{G_{1,2}} << \frac{c}{G_{ext}} = \tau_\infty$$

Only weak relationships hold, such as:

$$\sum_{i=1}^{N} \frac{1}{\tau_i^{(Local)}} = \sum_{k=1}^{N} \frac{1}{\tau_k} \qquad \min_{1\le i\le N}\left\{\tau_i^{(Local)}\right\} \le 2\tau_k \le 2\tau_\infty$$

Local vs terminal time constant paradox

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Thanks for your attention!

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