Appendix J

Methods for solving linearized networks in satellite thermal analysis

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Abstract

The presentation discusses a further development step of the Astrium in-house S/W tool TransFAST. This tool was originally developed in order to establish methods for calculation of transfer functions in the frequency domain, as was required for thermal analysis of the LISA missions. The mandatory first step of such type of analysis is the transformation of the classical thermal network to a standard linear control system by linearization of the radiative terms at a certain steady-state. As an extension of the existing tool, this linear control system shall be solved in the standard time domain.

Application of this type of analysis becomes important for all missions, where extremely demanding requirements on geometrical and thus thermo-elastic stability are involved. In such cases the deviations from a certain steady-state are small enough for performing thermal analysis on linearized systems. The major aim of such methods is to perform analyses with significantly less effort compared to the classical approach, but promising to deliver reasonable and even more accurate results.

Two different approaches for solving the linearized thermal network in the time domain are presented, the well-known ordinary differential equation (ODE) methods, and a quasi-analytical method, which splits the differential equations in a homogenous and an in-homogenous part. The major advantage of analytic solutions would be that no transient calculation for the whole time period is necessary. This is particularly suitable for problems where only a small number of specified time points are of interest. Also these time points can be selected w/o any limitation w.r.t. exactness of the solution, because the calculation requires only the analysis of a function, instead of solving an algorithm which gradually time step by time step creates the solution. This implies that a (final) steady-state solution caused by a certain disturbance can be directly calculated without having potential numerical problems as for a transient calculation. Results obtained by these two approaches are compared vice versa and with results calculated by the standard thermal analysis S/W.

Methods for Solving Linearized Networks in Satellite Thermal Analysis

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Overview

Methods for Solving Linearized Networks in Satellite Thermal Analysis

- Motivation & Methodology
- ODE Approach
- Quasi-analytical Approach
- Exemplary Results
- Summary & Outlook

All the space you need
— Page 2
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Motivation & Methodology

- Current & future science missions require ultra-stable S/C structures, with extremely demanding thermo-elastic stability requirements
- Thermal analysis accuracy has to be significantly improved
- Application examples:
  - → LISA aims to detect gravitational waves
  - → GAIA aims to create a precision 3-d star map of the galaxy
  - → Others …
- Perform thermal disturbance analysis for small deviations from the nominal state (i.e. standard thermal analysis steady-state solution)
- Linearization of the radiative terms of the heat balance equation, subsequent solution of the equation (linear control system now)
- Use standard programming environment (MATLAB)

All the space you need — Page 3
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Linear Control Methods

- Methods for application of linear control methods well established
  - → linearization of the heat balance equation
  - → frequency domain application (LISA, LISA Pathfinder)
    - → **d**irect **i**nversion of the **t**ransformed system matrix (DIT)
    - → **c**onditioned **e**valuation of the **f**requency response (CEF)

  see:
  Altenburg & Burkhardt: “A Software Tool Applying Linear Control Methods to Satellite Thermal Analysis” (last year workshop presentation)
- Significant advantages compared to standard methods:
  - → reduced computational & memory effort
  - → promises higher accuracy
- Extension of methods to (standard) time domain applications, e.g. Gaia
  - → ODE solvers, others …

All the space you need — Page 4
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ODE (Ordinary Differential Equation) Approach

- Idea: apply standard methods for solving the linear control system, e.g. ODE solvers
- Thermal system characteristics:

→ stiff by nature
→ only real and negative Eigenvalues (heat flow in one direction only)
→ large difference between biggest and smallest Eigenvalue (orders of magnitude)

- Stiff problems are often solved better by applying implicit numerical methods
- Extensive comparison of MATLAB built-in solvers and others (literature search) w.r.t. computational effort and accuracy done

↓

- MATLAB built-on solver ODE15s showed best performance

→ implicit numerical ordinary differential equation solver
→ works with backward differentiation formulas and numerical differentiation formulas, based on the differentiation of a Lagrange polynomial

$$\sum_{m=1}^{k} \frac{1}{m}\nabla^m y_{i+1} - h\bullet F(x_{i+1}, y_{i+1}) = 0 \qquad \sum_{m=1}^{k} \frac{1}{m}\nabla^m y_{i+1} - h\bullet F(x_{i+1}, y_{i+1}) - \kappa\gamma_k\left(y_{i+1} - y_{i+1}^{(0)}\right) = 0$$

- ODE15s selected as baseline solver for time domain analysis

All the space you need — Page 5
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Quasi-analytical Approach (1)

- Extended search for numerical solvers led to the idea to check also analytical methods
- Advantage of an analytical solution:
  → requires only the analysis of a function, instead of solving the complete algorithm
  → specific times of interest can be selected w/o any limitation w.r.t. exactness of the solution
  → steady-state solution can be directly calculated (particular part of the equation)

- Mathematical approach to solve a linear differential equation system

$$[\dot{x}] = [A_{DD}]\bullet[x] + [bu]$$

disturbance vector

- Split problem into a homogenous and a particular part ($x = x_h + x_p$)
  Solve the homogenous part with the standard approach for linear differential equation systems

$$[x_h] = c\bullet[v]\bullet e^{\lambda t} \xrightarrow{\text{Differentiation}} [\dot{x}_h] = \lambda\bullet c\bullet[v]\bullet e^{\lambda t}$$

- Apply standard approach and differential to homogenous part

$$\lambda\bullet c\bullet[v]\bullet e^{\lambda t} = [A_{DD}]\bullet c\bullet[v]\bullet e^{\lambda t} \xrightarrow{\text{Transformation}} \det([E]\bullet\lambda - [A_{DD}]) = 0$$

- Calculate Eigenvalues and Eigenvectors → homogenous solution

$$[x_h] = c_1\bullet[v_1]\bullet e^{\lambda_1\bullet t} + c_2\bullet[v_2]\bullet e^{\lambda_2\bullet t} + \ldots\ldots + c_n\bullet[v_n]\bullet e^{\lambda_n\bullet t}$$

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Quasi-analytical Approach (2)

- Particular solution depends on type of disturbance vector
- Example: constant disturbance vector *bu*
  → polynomial approach of first order (x_p = const, dx_p/dt = zero)
- Insert into differential equation and transform

$$[A_{DD}]\cdot[x_p] = -[bu]$$

- Combine homogenous and particular solution, solve the initial value problem by defining a logic initial value equal zero

$$[0] = [x_h(0)] + [x_p(0)]$$

$$[x_h(0)] = c_1\cdot[v_1] + c_2\cdot[v_2] + \ldots\ldots + c_n\cdot[v_n] = [V]\cdot[c]$$

- The system is transformed and the constants can be defined

$$[c] = -[V^{-1}]\cdot[x_p(0)]$$

- Extension to general disturbance vector (sine, Fourier series) under investigation

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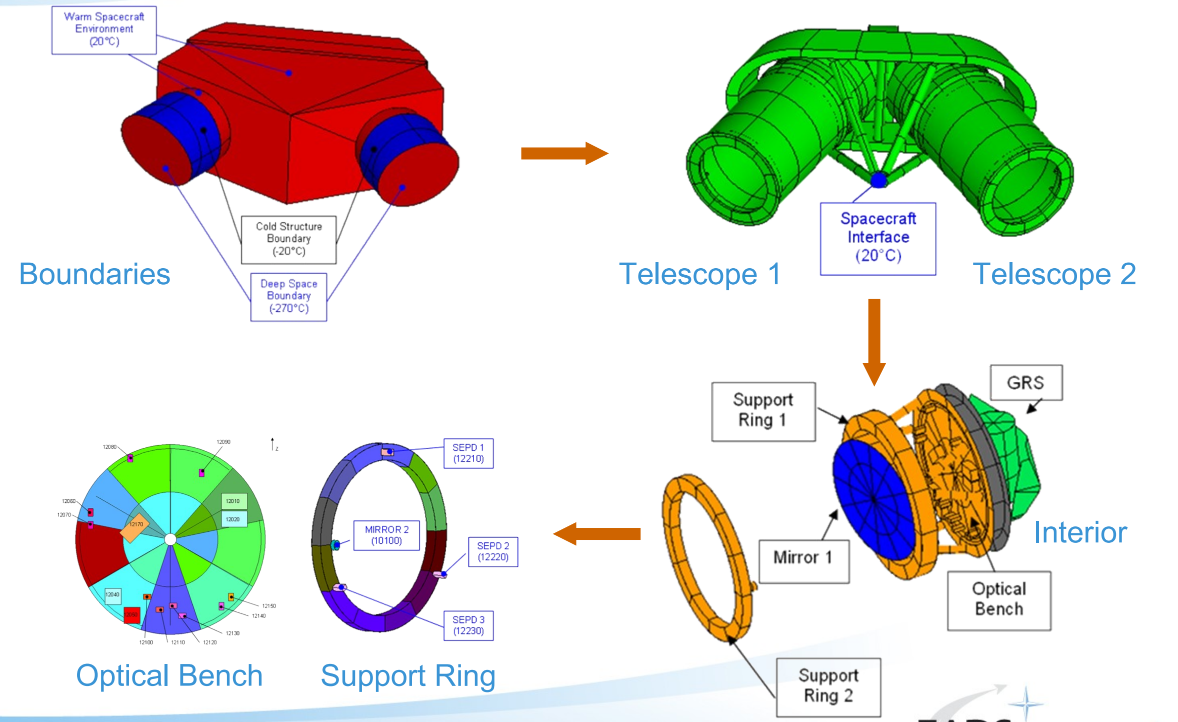
LISA Payload



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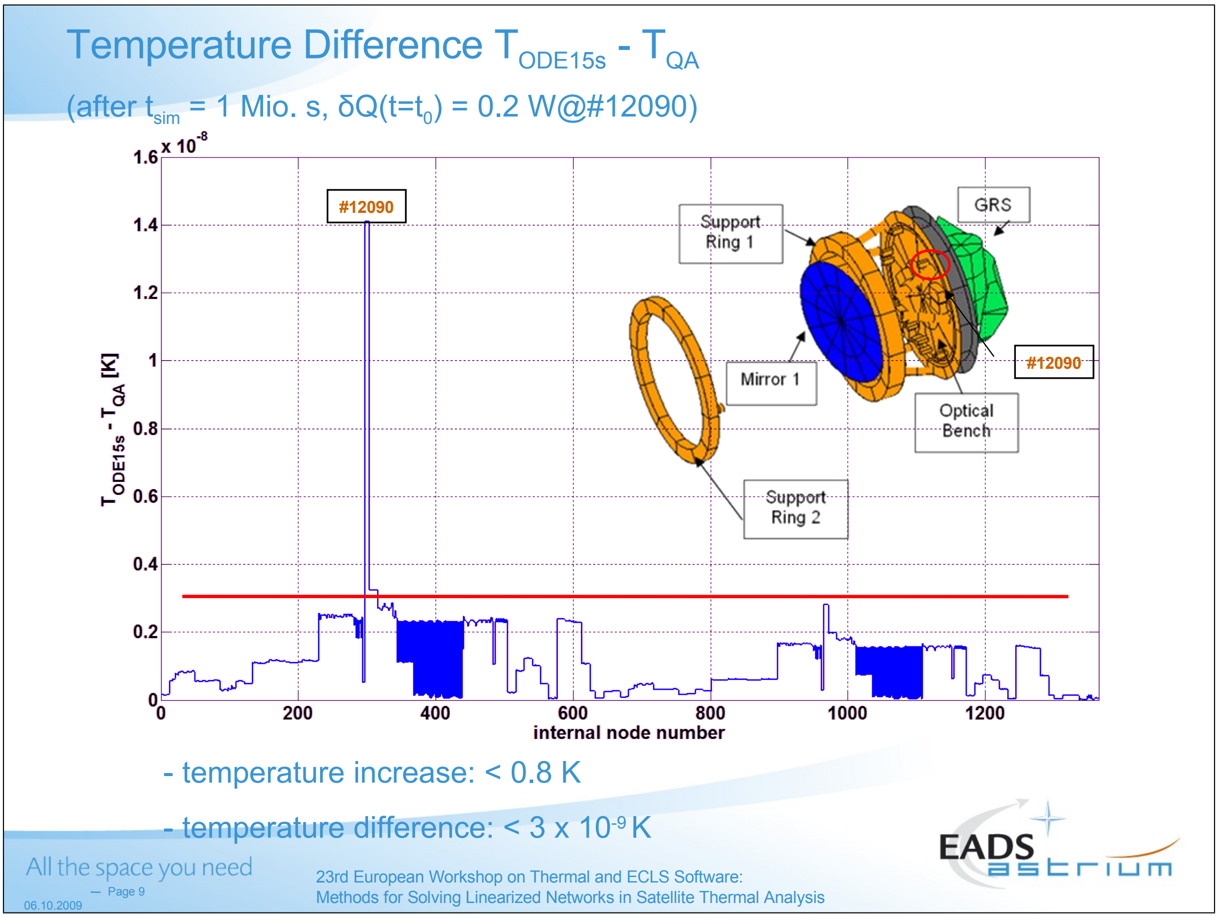
Temperature Difference T ODE15s - T QA
(after t sim = 1 Mio. s, δQ(t=t0) = 0.2 W@#12090)
x 10-8
TODE15s - TQA [K]
internal node number
#12090
Support Ring 1
GRS
Mirror 1
#12090
Optical Bench
Support Ring 2
- temperature increase: < 0.8 K
- temperature difference: < 3 x 10-9 K
All the space you need
Page 9
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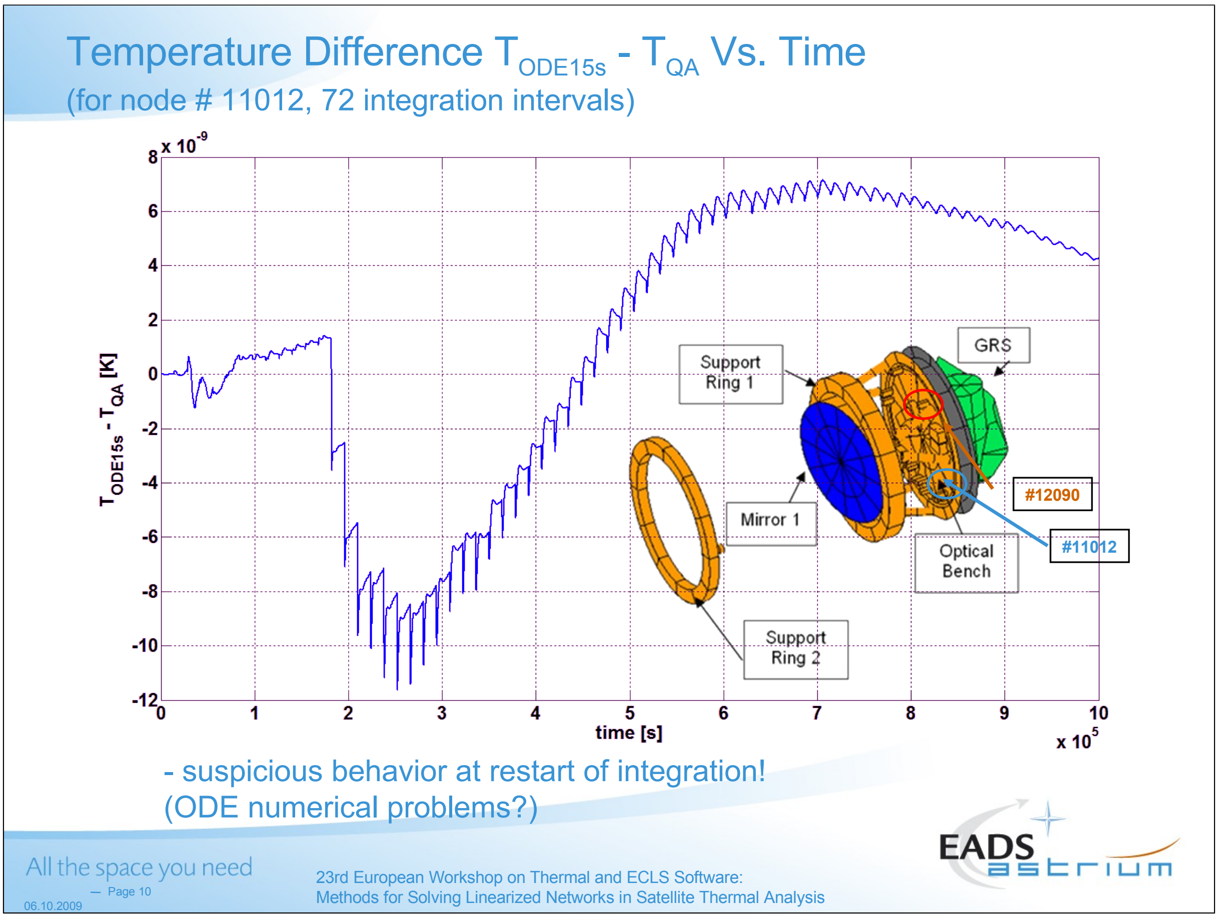
Temperature Difference T ODE15s - T QA Vs. Time
(for node # 11012, 72 integration intervals)
x 10-9
TODE15s - TQA [K]
time [s]
x 105
Support Ring 1
GRS
Mirror 1
#12090
#11012
Optical Bench
Support Ring 2
- suspicious behavior at restart of integration!
(ODE numerical problems?)
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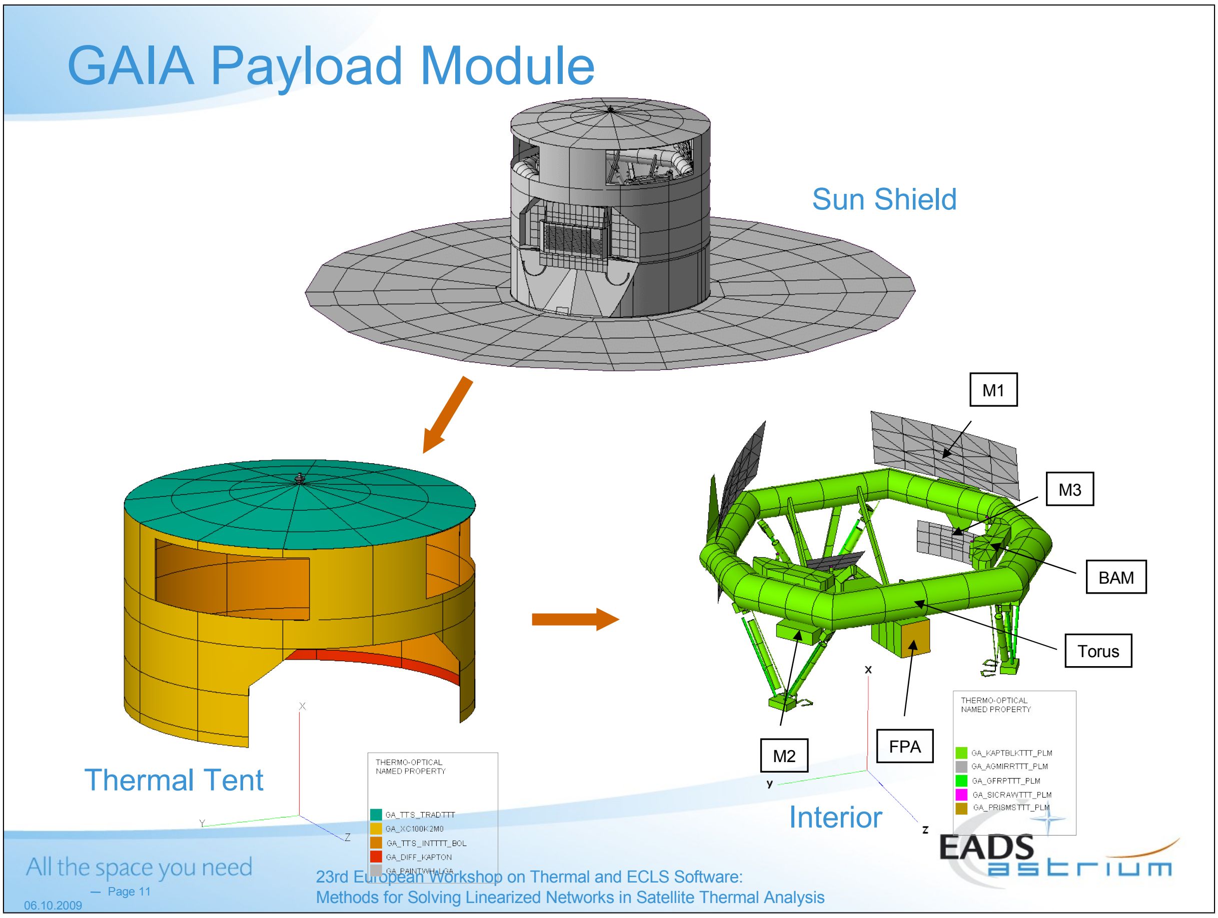
GAIA Payload Module
Sun Shield
Thermal Tent
Interior
M1
M3
BAM
Torus
M2
FPA
THERMO-OPTICAL NAMED PROPERTY
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— Page 11
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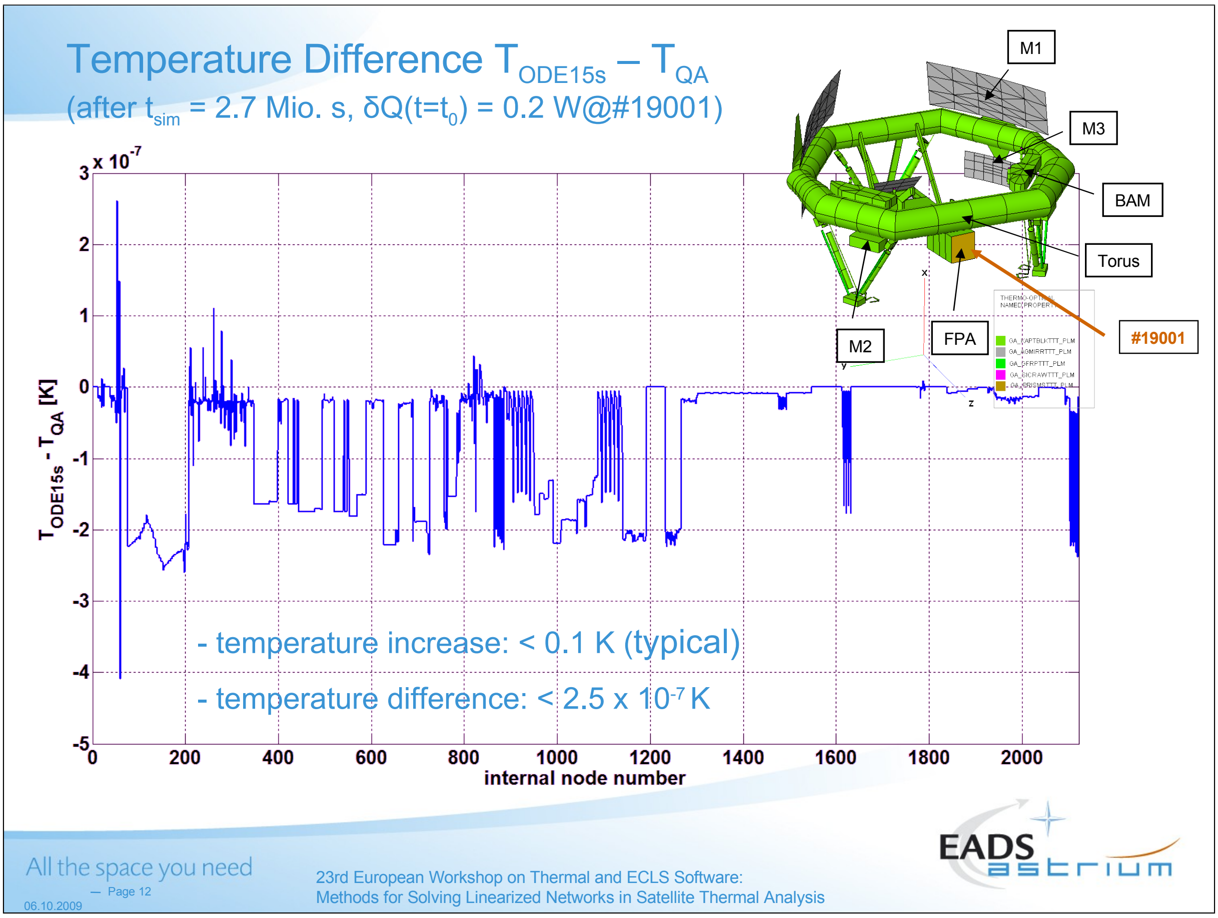
Temperature Difference $T_{ODE15s} - T_{QA}$
(after t_{sim} = 2.7 Mio. s, $\delta Q(t=t_0)$ = 0.2 W@#19001)
x 10^{-7}
$T_{ODE15s} - T_{QA}$ [K]
internal node number
- temperature increase: < 0.1 K (typical)
- temperature difference: < 2.5 x 10^{-7} K
M1
M3
BAM
Torus
M2
FPA
#19001
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— Page 12
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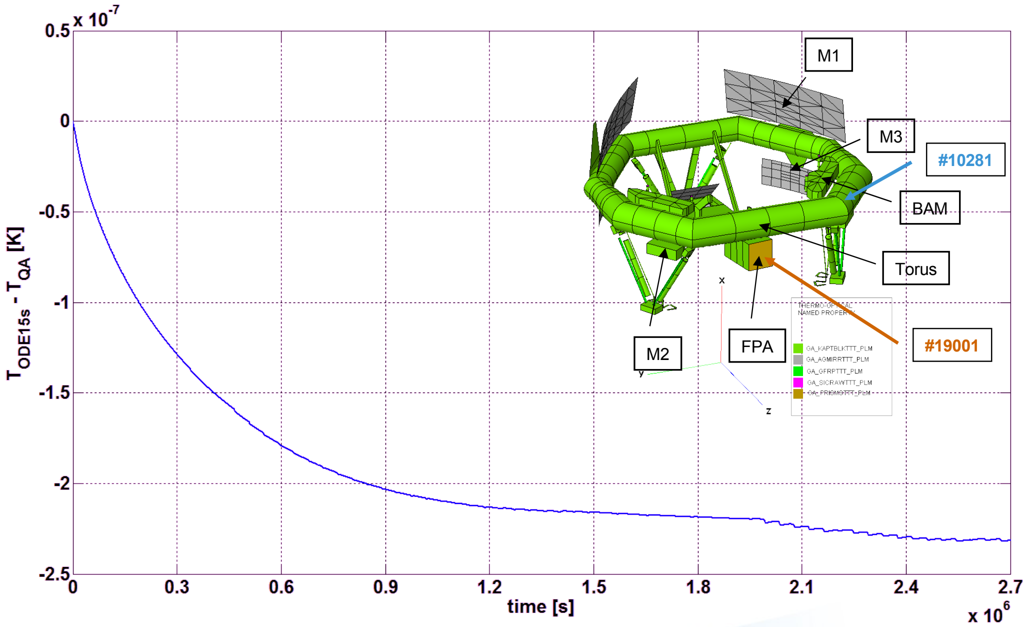
Temperature Difference T_{ODE15s} - T_{QA} Vs. Time

(for node # 10281)



- temperature difference drift → solutions diverge

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ODE vs. Quasi-analytical Approach (Summary)

- **LISA payload model**
  → very small temperature differences, ODE has some problems to resolve initial (step) input
  → temperature difference vs. time plots: no numerical drift
- **Gaia PLM model**
  → differences larger compared to LISA model, close to required absolute accuracy (μK range)
  → temperature difference vs. time plots: solutions diverge
- **Ordinary Differential Equation**
  → Pro: - capability to calculate any time-dependent perturbation function
  → Con: - needs to be run in small time interval steps (accuracy!)
  - difficult to identify the calculation error
- **Quasi-analytical approach**
  → Pro: - quasi steady-state calculation at any certain time of interest
  - in general faster
  → Con: - no guarantee to obtain a reasonable result for any type of perturbation
  - unknown accuracy for new particular solutions

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Comparison of Results – Standard Model vs. Linear Model

- Difference between linearized and non-linearized system

$$T^4 = \left(T_e + \delta T\right)^4 \cong 1 \cdot T_e^{\,4} + 4 \cdot T_e^{\,3} \cdot \delta T$$

- Approach for error estimation
  → calculation of a a steady-state solution in ESATAN
  → constant disturbance: new linear and nonlinear steady state calculation

$$\delta T_{nl} = T_{enl} - T_e \qquad \delta T_l = T_{el} - T_e$$

  → calculation of absolute and relative (scaled) differences

$$\delta T_{absdiff} = \delta T_{nl} - \delta T_l \qquad \delta T_{reldiff} = \frac{\delta T_{nl} - \delta T_l}{\delta T_{nl}}$$

- Expectations:
  → smaller relative error for smaller perturbations
  → smaller relative error at parts with less radiative heat exchange
  → non-linear solution should provide a cooler system (lower system energy)

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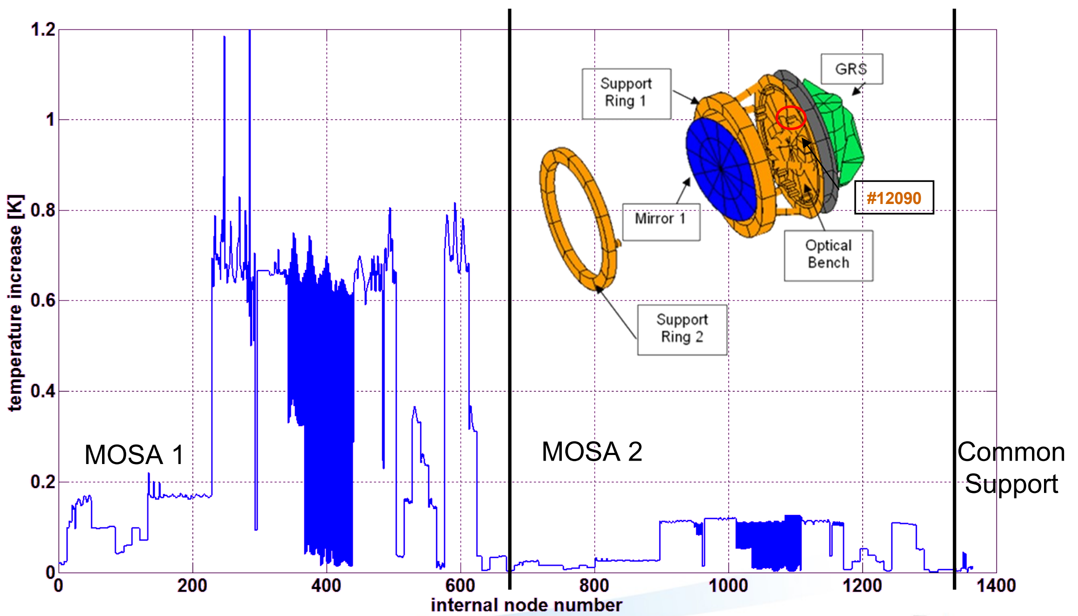
Temperature Increase for δQ =0.2 W @ # 12090 (Non-linear Solver)



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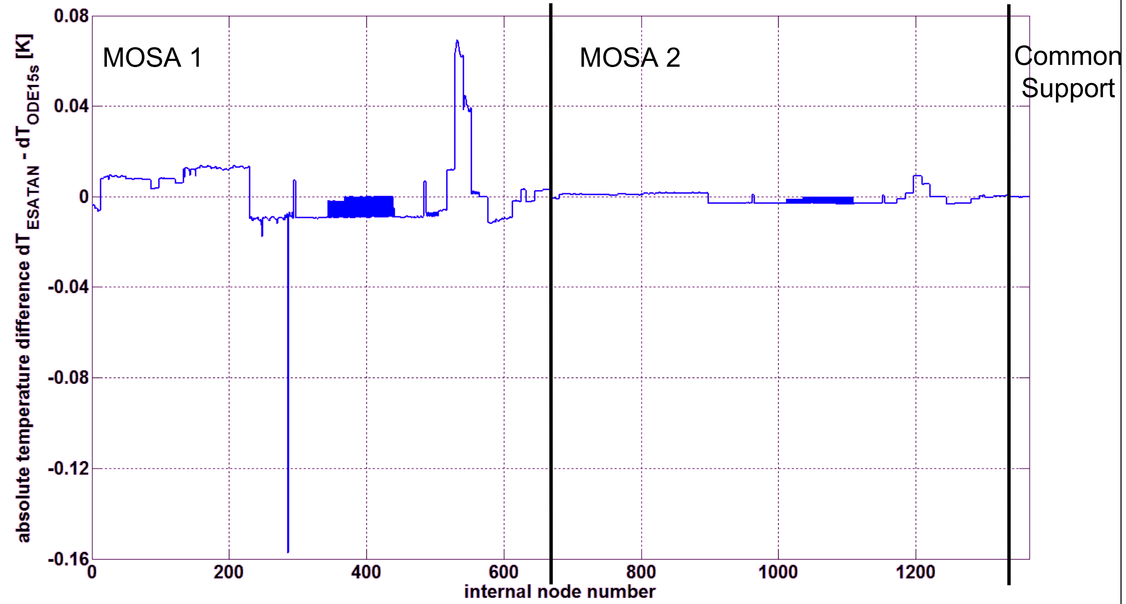
Absolute Temperature Difference Between Non-linear And Linear Solver, δQ =0.2 W @ # 12090



- temperature increase: < 0.8 K
- temperature difference: < 0.08 K (< 0.02 K typical)

All the space you need — Page 17
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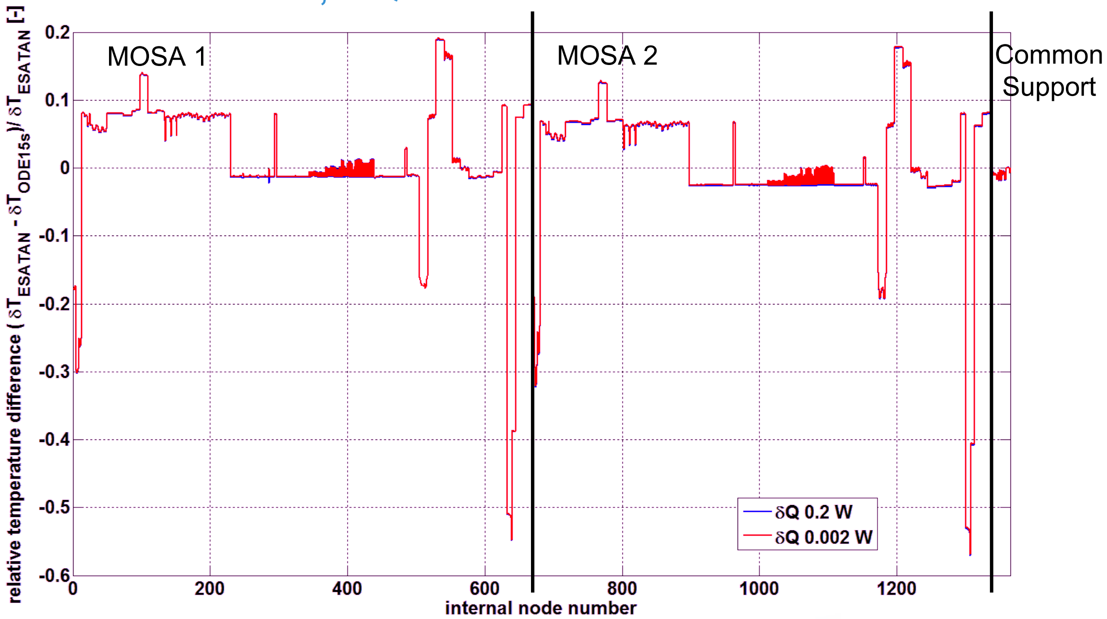
Relative Temperature Difference Between Non-linear And Linear Solver, δQ =0.2 & 0.002 W



MOSA 2 results similar to MOSA 1 results, independent from source of disturbance (@MOSA 1)!

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Comparison of Results – Standard model vs. Linear Model (Summary)

- Link between level of disturbance and relative temperature differences confirmed only at nodes close to source of disturbance
- At some locations: unacceptable large deviations (>50%)
- Deviations are nearly independent from the level of disturbance (two telescopes!)

- Explanation:
  → ESATAN model and linear (MATLAB) model parameters are not exactly identical
  → potential linearization errors much smaller than effects induced by small differences of system parameters
  It has to be ensured that the system description of both models (Cp, GL, GR, etc.) is exactly identical!
  (Small differences were acceptable for frequency domain application)

- Assess/resolve this problem by
  → avoiding zero-capacities (MLI, arithmetic nodes) in the ESATAN model
  → applying only double precision accuracy, (.000000000000 - .000000368961)

All the space you need — Page 19
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Summary & Outlook

- Methods for solving linearized thermal networks established
  → frequency domain (DIT & CEF)
  → time domain (ODE & QA)

- Non-linear vs. linear: discrepancies, but not linked to linearization

- Linear systems
  goal:
  establish verified and reliable methods for time domain analysis in order to solve high-accuracy problems
  further activities:
  → re-asses MATLAB internal memory problem (ODE, large models)
  → further work on quasi-analytical methods
  (general disturbance vector (sine, Fourier series))
  → implementation of (finally selected) methods in Astrium in-house tool TransFAST

- Solve standard non-linear thermal system in MATLAB
- Broader Literature research (solvers used by other disciplines)

All the space you need — Page 20
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Appendix - Linearization Approach

1) Re-arrangement of the thermal network matrices

$$CP_i \cdot \frac{dT_i}{dt} = \sum_{i,j=1}^{n}\left(\frac{\lambda_{i,j}\cdot A_{i,j}}{l_{i,j}}\cdot\left(T_j-T_i\right)\right)+\sum_{i,j=1}^{n}\left(\sigma\cdot\varepsilon_i\cdot A_i\left(T_j^4-T_i^4\right)\right)+Q_i \qquad diag[C]\cdot\left[\frac{dT}{dt}\right]=[K]\cdot[T]+[F]\cdot\left[T^4\right]+[Q]$$

1) Linearization of radiative terms around the equilibrium state

$$T^4=\left(T_e+\delta T\right)^4 \cong 1\cdot T_e^4+4\cdot T_e^3\cdot\delta T$$

1) Linear Control System
→ Distinguish node types

$$\left[\dot{x}\right]=[A]\cdot[x]+[B]\cdot[u]$$

$$[y]=[C]\cdot[x]+[D]\cdot[u]$$

Node Properties:
- Node number (N)
- External & internal heat sources (Q)
- Steady-State temperature (T)
- Heat capacity (CP)

Node Links:
- Heat conductance coefficient (GL)
- Heat radiation coefficient (GR)

Node Property:
- Heat capacity (CP)

[A]: Steady-State temperature (T), Heat capacity (CP), Heat conductance coefficient (GL), Heat radiation coefficient (GR)

[B]: Heat capacity (CP)

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