Analysis of Spacecraft Thermal Stability

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Objectives of Presentation

- A short study is just underway to assess the Phase A thermal stability analysis that has been made for the Laser Interferometer Space Antenna (LISA).

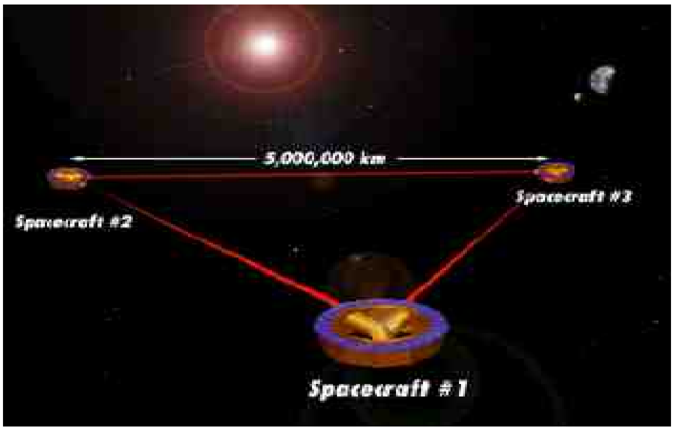
- The study aims to:
  - assess the validity of the standard thermal modelling approaches when investigating small temperature fluctuations.
  - recommend methods for improving analysis of spacecraft thermal stability.

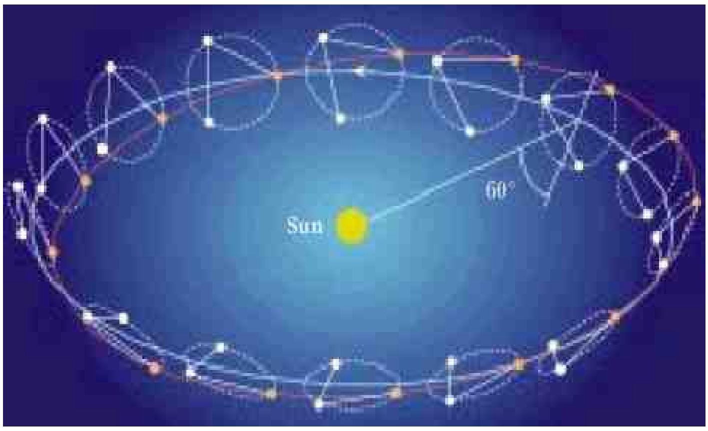
- The purpose of this presentation is therefore to describe the work in progress:
  - summarise the initial review of uncertainty and error sources.
  - place the review in the context of spacecraft thermal stability analysis.
  - invite ideas and suggestions.

LISA Mission Summary

- Gravitational wave detector based on a Space interferometer:
    - Measures changes in separation between free falling test masses to 10pm accuracy.
    - Six test masses on three spacecraft in an equilateral triangle formation.
- Major thermal disturbances are due to:
    - Variation in solar constant due to Sun’s normal modes of oscillation.
    - Fluctuations in electrical power dissipation.
- Thermal stability requirements:
    - Optical Bench temperature stability better than 1×10^{-6} K/Hz$^{1/2}$ at 1 $\times10^{-3}$ Hz.
    - Electronics temperature stability better than 1.2×10^{-3} K/Hz$^{1/2}$.
    - Telescope optical path length changes resulting from thermo-elastic effects < 40pm/Hz$^{1/2}$

LISA Mission Summary





Thermal Modelling

- The thermal response of the unit is assessed by applying fluctuations, at specific frequencies, due to:
    - Solar Constant.
    - Electrical power dissipations.

- The semi-amplitude of temperature fluctuations is obtained from the absolute temperature predictions (i.e., $\frac{1}{2}(T_{max} - T_{min})$).
    - numerical stability must be achieved

- Therefore, the thermal model must be able to identify temperature differences smaller than 2×10^{-6} K (the difference between the absolute temperature predictions).

Thermal Stability Analysis Issues

- Understand the sources of instability
    - numerical instability.
    - ‘real’ instability.

- How to select appropriate model parameters
    - time steps.
    - meshing density.

- What are the limits on the accuracy for stability analysis predictions?

The Three Main Sources of Numerical Error

Data Uncertainty

Differences between the thermal model and the final ‘as-built’ spacecraft. For example,

- Heat capacities.
- Radiative exchange-factors.
- Conductances.

Rounding

Computer approximation to the real number.

Truncation

Replacing the exact partial differential equations with finite difference approximations.

Data Uncertainty

- LISA thermal analysis is for Phase A - details of the final spacecraft configuration are not known.

- The data uncertainty can be reduced to some extent as the design is developed (e.g., through Phases B and C).

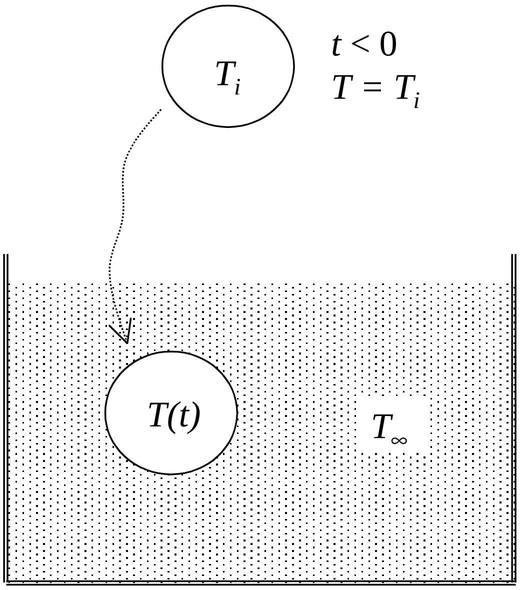
- Sensitivity analysis can be used to quantify these uncertainties.

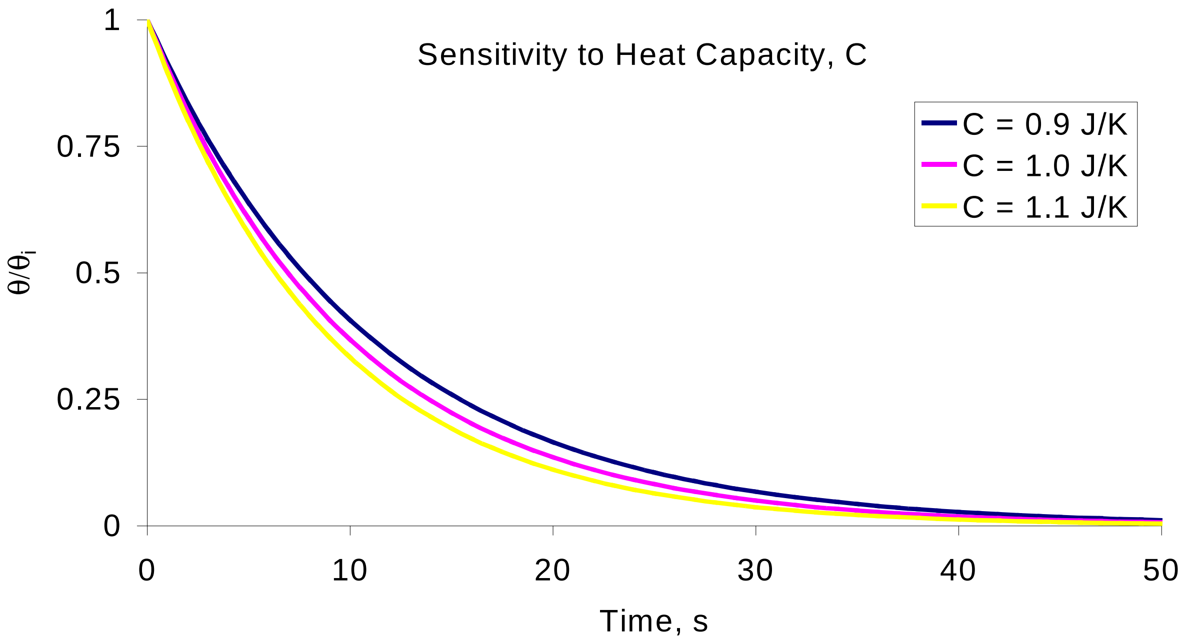
Data Uncertainty Example

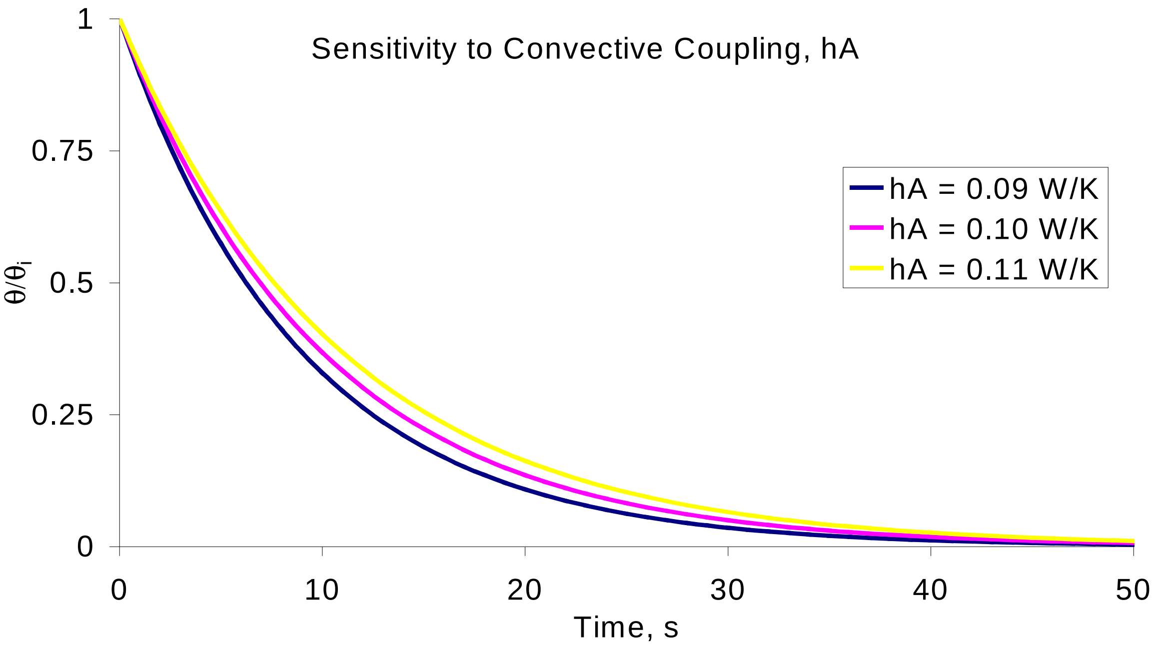
- Object at uniform initial temperature T_i
- Immersed in fluid at temperature T_∞ at t=0
- Convective coupling = hA = 0.1 W/K
- Heat capacity = $C = mc_p$ = 1.0 J/K

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hA}{C}\right)t\right]$$

$$\tau = \frac{C}{hA}$$







Rounding

- A characteristic of computer hardware.

- Floating point number system: $y = \pm m \times \beta^{e-t}$,
where m = mantissa; β = base; *e* = exponent range; *t* = bias.

- The sign, mantissa, and exponent range only are stored to represent floating point numbers.

- The two main floating point formats are:

Type	Size (bits)	Mantissa (bits)	Exponent (bits)	Unit roundoff	Range
Single	32	23+1	8	$\sim 6\times10^{-8}$	$10^{\pm38}$
Double	64	52+1	11	$\sim 1\times10^{-16}$	$10^{\pm308}$

- ESATAN solution routines use Double Precision

Accumulation of Rounding Errors

- Theoretically, in finite difference routines the rounding errors accumulate linearly with time.
  - the occurrence of rounding errors is not random.
- Minimising the time step will increase the rounding errors.
  - ‘larger’ time step is desirable.
- Cancellation
  - is when two nearly equal numbers are subtracted.
  - can exacerbate problems.
  - occasionally beneficial because errors are also cancelled.
  - avoid subtracting values that are in error.

Truncation

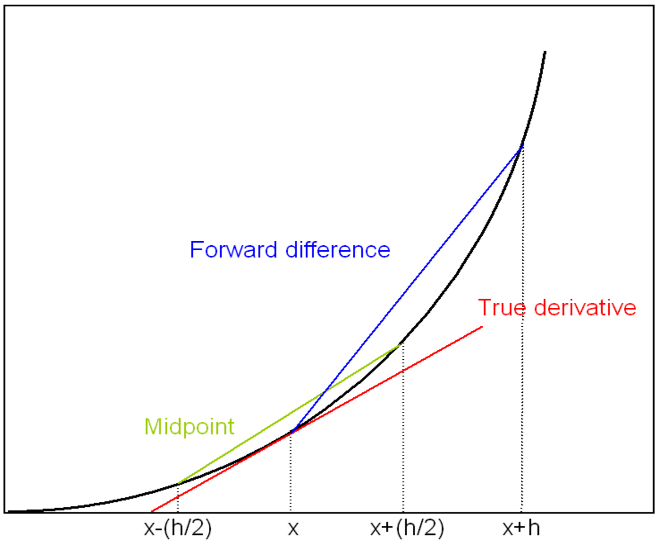
- Truncation error would exist even if a computer had perfect representation of numbers and no round-off error.
- Truncation errors are associated with the discrete approximation of a continuous quantity.
  - time-steps.
  - spatial discretisation.
- Minimising time step and refining mesh is desirable.
- Spatial discretisation is generally coarse in Phase A models and becomes finer, as appropriate, during Phases B and C.
- A fine mesh is desirable for predicting temperature gradients over low conductivity structures.

Truncation

- The ESATAN ‘SLFWBK’ solver was used for the LISA Phase A thermal analysis:
  - Crank-Nicholson forward-backward difference method.
  - centred on the midpoint of the time-step.
  - ‘implicit scheme’: simultaneous solution of temperatures at each time-step.
  - numerically stable for any size of time-step.
  - the time discretisation is ‘second order’ correct (i.e., proportional to the square of the time step):
    - halving the time-step reduces the truncation error by a factor of 4.

Midpoint Scheme

- The slope of the tangent to a function at the midpoint is a better approximation to the derivative than the ‘forward difference’ scheme

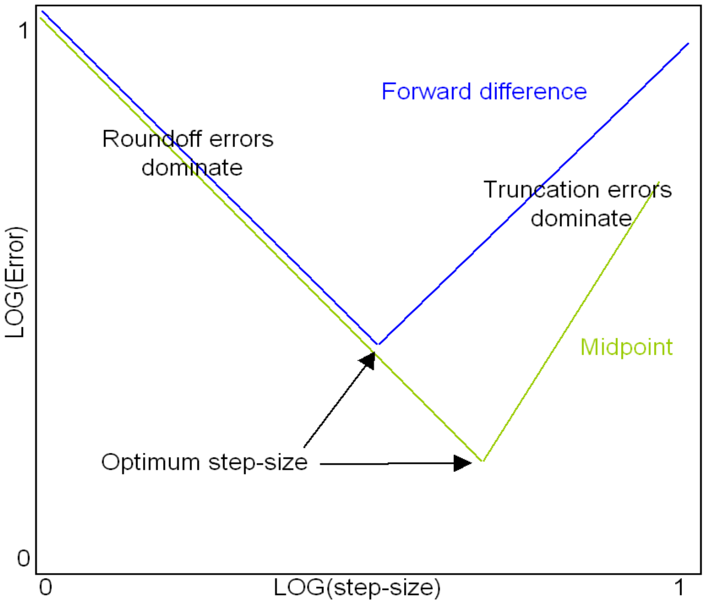


Influence of Step-Size on Truncation and Rounding

- Simply reducing the step-size as much as possible does not achieve the best accuracy.
  - truncation : minimise step-size.
  - rounding : ‘larger’ step-size.

- The optimal step-size therefore needs to be determined.

- Can the transient response be analysed to identify optimal step-size:
  - statistical.
  - noise.



Conclusions

- Three main sources of error that can influence thermal stability analysis:
  - data uncertainty.
  - truncation.
  - rounding.

- Planned work:
  - Complete review of error sources.
  - Assess hardware/software characteristics.
  - Develop methodology for setting up and undertaking thermal stability analysis.
  - Is present analysis capable of assessing the LISA thermal requirements?